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MISCELLANEA

PROCESS OF DEPOSITION OF A MATERIAL ON THE SURFACE OF A ROTATING BODY

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A mathematical model for estimating the thickness of a coating deposited on the surface of a rotating body having a circular symmetry is proposed. A method has been developed for determining the law of control of the mass flow of a deposited material for the purpose of obtaining coatings homogeneous in thickness. It is shown that, in the case of deposition of a coating on the surface of a circular disk, the optimum mass flow of a deposited material is defined by a linear function.

At present, spraying is widely used for deposition of coatings on different bodies. An important parameter of the coatings obtained by this method is the uniformity of distribution of a material over the surface of a body. The homogeneity of functional coatings, in particular metal and nonmetal ones, deposited on a body by thermal spraying is characterized by the difference between their thicknesses at different points of the surface of the body. Spraying is also used for deposition of solutions and suspensions containing a dispersed material. For example, diamond micropowders are deposited by spraying on the working surface of cutting disks used for grinding and polishing of diamond single crystals and other superhard materials [1, 2]. The uniformity of deposition of such coatings is characterized by the density distribution of the dispersed-material particles over the surface of the body. To obtain homogeneous coatings, calling for a minimum thermal treatment, and heterogeneous coatings with a uniform distribution of the dispersed material, it is necessary to have efficient methods for numerical simulation of the deposition process because the empirical optimization of this process on the basis of its physical simulation is very expensive and time consuming. In this case, it is necessary to consider a deposition process with nonstationary parameters because, in the process of deposition with stationary parameters, a deposited material is distributed nonuniformly over the surface of a body [2, 3]. In [3, 4], the possibility of obtaining uniform coatings in the case where the kinematic parameters of the deposition process (the trajectory and velocity of movement of a splitting apparatus) are controlled was considered. However, the kinematic parameters of this process are controlled with the use of complex manipulators, an error in the movement of whose working unit leads to the appearance of an error in determining the distribution of the parameters of the coating. Below it will be shown that the uniformity of deposition of coatings can be controlled by changing the mass flow of the deposited material. We will define the deposition of coatings with the use of a mathematical model based on the model of deposition of a diamond-containing suspension on the surface of a cutting disk, developed by us in [2].

In [2], it was assumed that in the case where the cutting disk is not moved relative to the nozzle of the spraying apparatus, the rate of deposition of diamond particles is constant throughout the deposition region. According to the data of [3], the rate of deposition of the coating by thermal spraying is defined most adequately by the Gaussian function

$$v_{\mathrm{d}}(\delta) = v_{\mathrm{d}0} \exp\left(-\delta^2/\sigma^2\right) .$$

The assumption used in [2] was verified in the following experiment.

An alcoholic solution of a phenolformaldehyde adhesive (three volume parts of ethyl alcohol and one volume part of the adhesive), used for the deposition of a diamond-containing suspension, was colored an intense green by the

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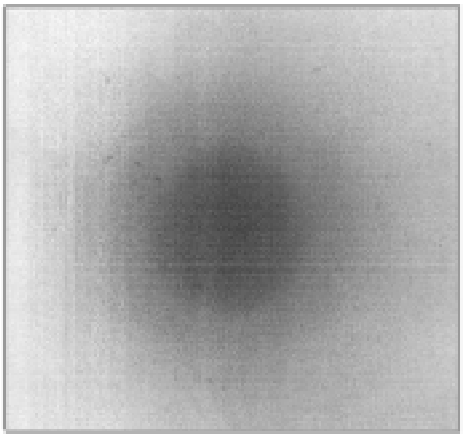

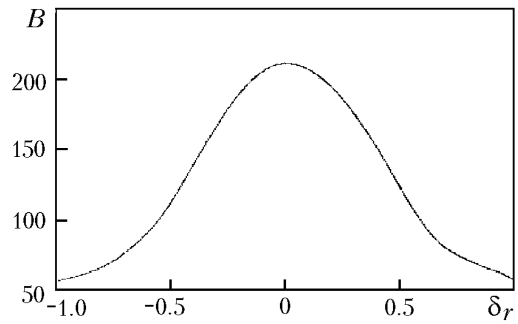

Fig. 1. Photography of a spraying spot on the surface of a body.

Fig. 2. Distribution of luminance at the cross section of the spraying-spot image.

Viridus Nitentis alcoholic solution and deposited, using an injection spraying system, on the surface of an acrylic-plastic sample. The spraying spot (Fig. 1) obtained using a digital photographic camera was processed on a computer with the use of the Mathcad computer-algebra program.

A halftone image processed with this program was represented in the form of a matrix with integral elements B taking values from 0 to 255 and characterizing the luminance of pixels. A pixel to which corresponds the largest element of the matrix was used as the conditional center of the image. It should be noted that the conditional center of the image of the spraying region, determined in this way, cannot be coincident with the actual center of this region — the point of intersection of the axis of the nozzle of a spraying apparatus with the surface of a sample. This is explained by the existence of local deviations in the luminance of the image, which are caused by the apparatus noises or represent the images of dye particles or foreign-impurity particles. Figure 2 shows the luminance distribution at the cross section of an image, passing through its conditional center.

This graph was constructed by the data smoothed with the use of the least-square technique. It represents the distribution of the rate of deposition of the material over the spraying region. This distribution can be approximated using a Gaussian function, which points to the fact that the model proposed in [2] should be verified. The experimental Gaussian distribution of the material is in good agreement with the data on the coloration by spraying, obtained in [5].

We now consider the process of deposition of a homogeneous functional coating on the plane surface of a rotating body having a cyclic symmetry. Since, in practice, the rate of deposition of a coating decreases to zero at any finite distance from the center of the spraying region, we will assume that this rate is defined by a Gaussian function only at the center of a circular zone of radius r and is equal to zero beyond this zone. The effective radius r of the spraying region will be determined from the condition $v_{\mathrm{d}}(r) \geq \varepsilon d_{\mathrm{d0}}$ ($\varepsilon = 5 \cdot 10^{-3}$ in the examples described below). In the case where the body and the nozzle of the spraying apparatus are moved relative to each other, the rate of deposition of the coating at an arbitrary point of the body surface with coordinate ρ will be dependent on time and defined as

$$v_{\mathrm{d}}(\rho, t) = v_{\mathrm{d0}} \exp\left(-\delta^2(\rho, t)/\sigma^2\right), \tag{1}$$

where the distance from the center of the spraying region to the indicated point will be determined by the cosine theorem

$$\delta^2(\rho, t) = (R - r - v_{\mathrm{g}}t)^2 + \rho^2 - 2\rho\,(R - r - v_{\mathrm{g}}t)\cos\omega t\,.$$

Here, it is assumed that the angular coordinate of the spraying region is coincident with the angular coordinate of the point being considered.

If the condition $\pi v_g/\omega r << 1$ is fulfilled, the dependence of the thickness of the coating h on the angular coordinate can be disregarded and this thickness can be assumed to be a function of only the radial coordinate ρ:

$$h(\rho) = \sum_{j=1}^{M} \int_{\tau_{j1}(\rho)}^{\tau_{j2}(\rho)} v_d(\rho, t)\, dt\,. \tag{2}$$

For the surface point with a coordinate ρ_0, the integral appearing in expression (2) takes nonzero values only at certain values of the index j; however, to represent this expression in the general form, which could be used at any values of the variable ρ, the summation should be performed over all possible values of j.

Let us consider the possibility of obtaining a coating homogeneous in thickness by changing the mass rate of flow μ of the deposited material. In the case of a nonstationary mass flow of the deposited material, the amplitude of function (1) will depend on the value of this flow and, consequently, on the time

$$v_{d0} = v_{d0}(\mu(t))\,.$$

On the assumption that the flow rate of the deposited material has no influence on the root-mean-square deviation of the rate of deposition of a coating (1), the flow rate of the deposited material, in the stationary case, will be related to the deposition rate by the relation

$$\mu = 2\pi\gamma \int_0^{+\infty} v_d(\delta)\, \delta d\delta\,.$$

Integration gives the following expression for the amplitude of the deposition-rate function:

$$v_{d0}(\mu) = \frac{\mu}{\pi\gamma\sigma^2}\,. \tag{3}$$

Thus, the amplitude of the deposition-rate function is in direct proportion to the flow rate of the deposited material.

In the case where the flow rate of the deposited material remains practically constant for the time $\tau_{j2}(\rho) - \tau_{j1}(\rho)$, expression (2) can be represented in the following form:

$$h(\rho) = \sum_{j=1}^{M} v_{d0j} \int_{\tau_{j1}(\rho)}^{\tau_{j2}(\rho)} \exp\left(-\delta^2(\rho, t)/\sigma^2\right) dt\,, \tag{4}$$

where $v_{d0j} = v_{d0}(\mu_j)$ and $\mu_j = \mu(2\pi(j-1)/\omega)$ is the flow rate of the material at the instant of time corresponding to the $j - 1$ complete revolutions of the body.

Equation (4) represents the main equation of the model of the deposition process. We will discretize it with respect to time. For this purpose, the total time T of movement of the spraying apparatus along the radius of the body will be divided into N_1 equal time intervals $\Delta t = (R - 2r)/v_g N_1$. In this case, denoting the integrand in expression (4) as $g(\rho, t)$ and using the quadrature formula of triangles for calculating the integral, we obtain

$$h(\rho) = \Delta t \sum_{j=1}^{M} v_{d0j} \sum_{i=0}^{M_{j2}(\rho)-M_{j1}(\rho)} g\left(\rho, (M_{j1}(\rho) + i)\, \Delta t\right) = \Delta t \sum_{j=1}^{M} v_{d0j} G_j(\rho)\,, \tag{5}$$

where $M_{j1}(\rho) = \dfrac{\tau_{j1}(\rho)}{\Delta t} + 1$ and $M_{j2}(\rho) = \dfrac{\tau_{j2}(\rho)}{\Delta t}$.

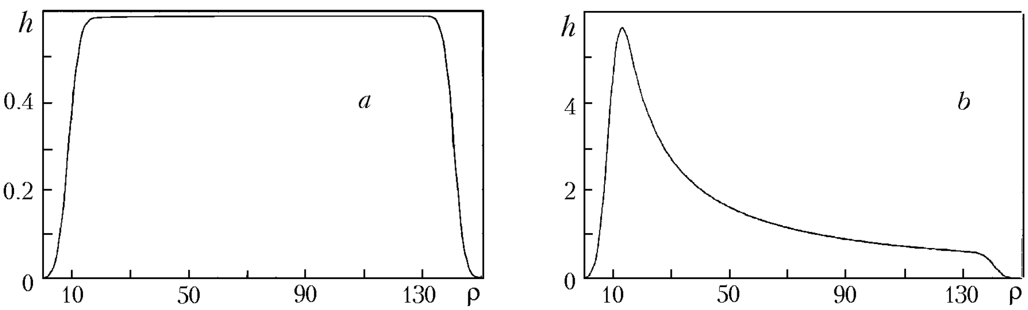

Fig. 3. Dependence of the thickness of the coating on its radial coordinate in the case where the deposition was performed with a nonstationary mass flow of the deposited material (a) and in the case of deposition with stationary parameters (b). h, μm; ρ, mm.

Assuming that the thickness of the coating is calculated at a finite number of points, we will discretize the model of the deposition process with respect to the space coordinate. The discretization step will be defined as $\Delta\rho = 2\pi v_g/\omega N_2$, where the number N_2 determines the discretization frequency. In this case, the coordinates of the discretization points will be determined from the expression

$$\rho_k = R - k\Delta\rho \ , \quad k = \overline{0, N_3} \ ,$$

where $N_3 = \dfrac{R}{\Delta\rho}$.

The spatial discretization of expression (5) gives

$$h\,(\rho_k) = \Delta t \sum_{j=j_{\min}(k)}^{j_{\max}(k)} v_{d0j} G_{j,k} \ , \tag{6}$$

where $G_{j,k} = G_j(\rho_k)$ and $j_{\min}(k)$ and $j_{\max}(k)$ are the minimum and maximum values of j, for which the condition $G_{j,k} \neq 0$ is fulfilled.

According to expression (6), the calculation of spatial changes in the function $h(\rho)$ reduces to the weighted summation of the changes in the function $G_j(\rho)$. In this case, the weight factors characterize the change in the amplitude of the deposition-rate function arising due to the dependence of the flow rate of the deposited material on the time. This allows the conclusion that the problem on determination of the law of control of the flow rate of the deposited material that would provide the deposition of coatings homogeneous in thickness can be reduced to the construction of a system of weight factors, providing the constancy of the sum involved on the right side of expression (6), for a large number (as large as possible) of k values. The indicated condition is provided by the weight factors determined in the following way:

$$v_{d0j} = \begin{cases} \dfrac{\max\limits_k G_{2,k}}{\max\limits_k G_{j,k}} \ , & 3 \le j \le M-1 \ ; \\[2ex] 1 \ , & j = 1 \quad \text{or} \quad j = 2 \ ; \\[2ex] \dfrac{\max\limits_k G_{2,k}}{\max\limits_k G_{M-1,k}} \ , & j = M \ . \end{cases} \tag{7}$$

Below we present numerical examples for the case of a circular disk.

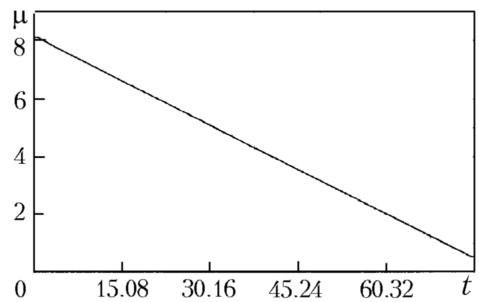

Fig. 4. Dependence of the mass flow of the deposited material on time in the case where a coating homogeneous in thickness is formed. μ, mg/sec; t, sec.

Figure 3a shows the dependence of the thickness of a coating on the radial coordinate in the case where the flow rate of the deposited material changes by the law determined by the weight factors (7). For comparison, an analogous dependence obtained for a stationary deposition is shown in Fig. 3b.

The calculations were carried out using a special program written in the Mathcad language for the following parameters of the problem: radius of the body R = 150 mm, rate of rotation of the body ω = 10 rad/sec, velocity of movement of the spraying apparatus v_g = 1.75 mm/sec, amplitude of the deposition-rate function v_{d0} = 0.017 mm/sec, root-mean-square deviation of the deposition-rate function σ = 4.127 mm, and density of the deposited material γ = 8.9 mg/mm^3 (nickel).

The calculated value of the effective radius r of the spraying region is equal to 9.5 mm.

The amplitude and roof-mean-square deviation of the deposition-rate function correspond to the literature data [3].

Figure 3a shows that, in the case where the flow rate of the deposited material is variable, the thickness of the deposited coating is constant practically along the whole radius of the body, excepting the regions in which the edge effect arises when the spraying apparatus begins to move or when it stops. The width of the regions where the edge effect arises is approximately equal to r. In the case of deposition with stationary parameters (Fig. 3b), the thickness of the coating increases as the center of the body is approached and then decreases to zero. The peripheral edge effect disappears when the spraying-region boundary is beyond the outer boundary of the body at the initial instant of movement of the spraying apparatus. The deposition becomes more uniform at the center of the body when this center is partially or completely intersected by the deposition region; however, a completely homogeneous coating is not obtained in this case. Attempts to provide a uniform deposition at the center of the body by changing the velocity of movement of the spraying apparatus and by displacement of the trajectory of its movement relative to the center of the body were made in [3]. However, despite the fact that the deposition became much more uniform, the coatings deposited at all the values of the deposition parameters being studied were not homogeneous in thickness at their center. It should be noted that, in the case where a body with a circular symmetry has a central hole, the edge effect is cancelled because the boundary of the spraying region is beyond the boundary of this hole.

The changes in the function defining the change in the flow rate of the deposited material with time were calculated from the known values of the weight factors with the use of relation (3). The time dependence of the flow rate of the deposited material, constructed by these data, is shown in Fig. 4. The initial value of the flow rate of the deposited material was determined from (3). This graph shows that the mass flow of the deposited material, determining the distribution of the thickness of the coating along its radius, shown in Fig. 3a, is defined by a linear function.

The accuracy of the calculations was verified by the fulfillment of the mass-conservation law:

$$2\pi\gamma \int_0^R h(\rho)\, \rho d\rho = \int_0^T \mu(t)\, dt\,.$$

The values of integrals calculated by the quadrature formula of trapezoids were equal accurate to 0.368%.

The model developed allows one to define the deposition of a coating on a curvilinear surface with the use of coefficients determining the influence of the angle between the axis of the spraying plume and the normal to the surface of the body on the rate of deposition of the coatings.

The method proposed for control of the uniformity of deposition of coatings on bodies can be easily realized with the use of known weighing apparatus.

The deposition of solutions and suspensions, containing a dispersed material, can be performed with a system used for deposition of diamond-containing suspensions on the working surface of cutting disks [6]. This system consists of a vessel with a suspension, moved by a mechanical mixer or as a result of bubbling, and a piston weigher, into the underplunger space of which a definite amount of the suspension is filled from the vessel before this suspension is deposited on the surface of the disk. The stock of the weigher moves over the former in the process of deposition and, in so doing, provides a differential feed of the suspension to different regions of the disk, located at different distances from its center.

In the case of thermal spraying, it is necessary to solve the problem of control of the mass flow of powder materials in real time. This problem arises when apparatus for direct laser deposition of functional-gradient coatings are designed. In [7], this problem was solved with the use of an auger weigher, in which the flow rate of the material is controlled by changing the rotational velocity of the auger with the use of a step drive.

Thus, the method proposed for simulation of the deposition of coatings on plane and curvilinear surfaces and programs based on this method allow industrial engineers to decrease the time of their work on simulation of deposition processes.

NOTATION

B, luminance of the image, arb. units; $g(\rho, t)$, integrand; $G_j(\rho)$, function obtained as a result of approximate integration of the function $g(\rho, t)$ over the time interval (τ_{j1}, τ_{j2}); h, thickness of the coating, m; M, number of intersections of the radial cross section of the surface of the body by the spraying region; M_{j1} and M_{j2}, numbers of discretization intervals at the instants of time τ_{j1} and τ_{j2}; N_1, number of time-discretization intervals; N_2, integer determining the frequency of the spatial discretization; N_3, number of spatial-discretization intervals; r, effective radius of the spraying region, m; R, radius of the body, m; t, time variable, sec; T, total time of spraying, sec; v_{d}, rate of deposition of the coating, m/sec; $v_{\mathrm{d}0}$, rate of deposition of the coating at the center of the spraying region, m/sec; v_{g}, velocity of movement of the spraying apparatus, m/sec; γ, density of the deposited material, kg/m^3; δ, distance from the center of the spraying region to the surface point being considered, m; δ_r, relative distance from the center of the spraying region to the point being considered; Δt, time-discretization step, sec; $\Delta\rho$, spatial-discretization step, m; ε, series expansion parameter; μ, mass flow of the deposited material, kg/sec; ρ, radial coordinate of a surface point, m; σ, root-mean-square deviation of the deposition-rate function, m; τ_{j1} and τ_{j2}, instants of time at which the point being considered enters the spraying region and leaves it at the jth intersection of the cross section being considered by this region, sec; ω, angular rotational velocity of the body, rad/sec. Subscripts: d, deposition; g, spraying apparatus; j, number of intersection of the radial cross section being considered of the surface of the body by the spraying region; k, number of spatial change in the function; 0, initial value (for the spraying-region center); max, maximum; min, minimum.

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